

Polarized Light and its Interaction
with Modulating Devices

-a Methodology Review-

J.C. Kemp



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J.C. Kemp *

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* Department of Physics, University of Oregon
Eugene, Oregon 97403-1274

Polarized Light and Its Interaction With Modulating Devices

Introduction

My first purpose here is to give a simple methodology for characterizing and analyzing polarized light. The main method will involve merely vectors, to represent the transverse electric fields -- generalized to the powerful method of complex vectors. Except for one illustration, I will avoid matrix methods (Jones, Mueller matrices), which are used in many texts and articles but which are unnecessarily cumbersome for many problems. A second purpose is to give a general description of polarized light, both coherent (monochromatic) and incoherent, and to describe a couple of useful theorems. Here I define the Stokes parameters, and show their relationship to the vector fields.

With this background I will discuss polarization modulators -- particularly so-called birefringence modulators -- and their use in measuring and transforming polarized light. A couple of examples using the HINDS International PEMTM (photoelastic modulators) will be covered in detail. In order to illustrate the power of the PEM-based approach, a last topic will be an interesting application of modulator techniques to a special problem, that of shifting a light beam's frequency.

A. Monochromatic polarized beam: Complex vector representation.

A single-frequency beam, with angular frequency $\omega = 2\pi\nu = 2\pi c/\lambda$, is shown in Fig. 1 (a). In a transverse stationary plane xy , the electric (E) vector generally describes an ellipse in time -- the wave is elliptically polarized. A compact way to describe this is by

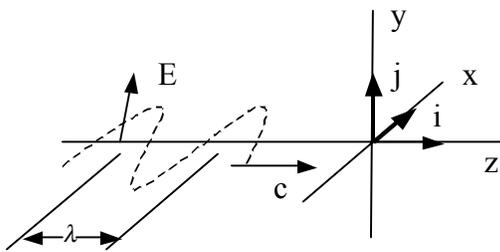


FIG. 1(a)

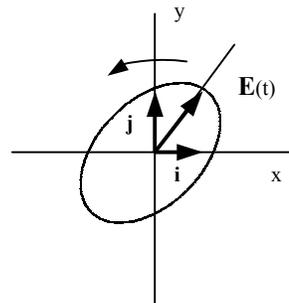


FIG. 1(b)

using a complex vector $\mathbf{E}(t)$:

$$\mathbf{E}(t) = (a\mathbf{i} + b\mathbf{j}) e^{-i\omega t} \quad (1)$$

where \mathbf{i}, \mathbf{j} are unit vectors along x, y ; and a, b are complex numbers. We write $a = a_1 + ia_2$ and $b = b_1 + ib_2$ where a_1, a_2, b_1, b_2 are all real numbers. Now, in the complex vector formalism we define the real or "physical" electric field as the real part of \mathbf{E} :

$$\mathbf{E}(phys) = \text{Re}\{\mathbf{E}\} = \mathbf{i}(a_1 \cos \omega t + a_2 \sin \omega t) + \mathbf{j}(b_1 \cos \omega t + b_2 \sin \omega t) \quad (2)$$

Using electric phase angles $\phi_x = \arctan(a_2/a_1)$ and $\phi_y = \arctan(b_2/b_1)$, this can also be written:

$$\mathbf{E}(phys) = \mathbf{i} |a| \cos(\omega t + \phi_x) + \mathbf{j} |b| \cos(\omega t + \phi_y) \quad (2')$$

where $|a| = \sqrt{a_1^2 + a_2^2}$ and $|b| = \sqrt{b_1^2 + b_2^2}$.

Examples: A circularly polarized wave rotating counterclockwise is:

$$\mathbf{E} = (\mathbf{i} + i\mathbf{j}) e^{-i\omega t} \quad (3)$$

for which $\mathbf{E}(phys) = \mathbf{i} \cos \omega t + \mathbf{j} \sin \omega t$

A linearly polarized wave at 45° orientation is merely:

$$\mathbf{E} = (\mathbf{i} + \mathbf{j}) e^{-i\omega t}, \quad \text{where } \mathbf{E}(phys) = (\mathbf{i} + \mathbf{j}) \cos \omega t. \quad (4)$$

Why use these "complex vectors"? Because it greatly economizes on the algebra we need to describe polarized light as it interacts with optical devices. In (1) we need only to keep track of the two (complex) numbers a, b . The time variation factor $\exp(-i\omega t)$ is always merely a common factor which rides along through all the calculations. In the latter we can often omit this time factor and put it back in only at the end of the calculation, when

we need the physically measurable fields. By contrast, if we work only with the physical field (2), we must carry along the four quantities $(a_1 a_2 b_1 b_2)$ as well as the separate cosine and sine time factors. (Alternatively if we work with the form (2') we must carry along the awkward phases ϕ_x, ϕ_y , which change as the light passes through a device.)

Note that the intensity of the wave is given by:

$$I = \overline{|\mathbf{E}(phys)|^2} = 1/2 \mathbf{E} \cdot \mathbf{E}^* = 1/2 (|a|^2 + |b|^2)$$

where the bar means averaging over one (or many) optical cycles. The factor 1/2 comes

from $\overline{\cos^2 \omega t} = \overline{\sin^2 \omega t} = 1/2$. Since we usually care only about ratios of intensities, for brevity I will drop the 1/2 here and define $I = \mathbf{E} \cdot \mathbf{E}^*$.

B. Stokes parameters.

The polarized nature of a light beam can normally be ascertained only by measuring the relative intensities of the beam after it is passed through certain devices, such as polarizing prisms or films, and wave plates. A standard set of such measures gives the four so-called Stokes parameters:

I = total intensity of incident beam.

$$Q = I_x - I_y$$

$$U = I_{x'} - I_{y'}$$

$$V = I_+ - I_-$$

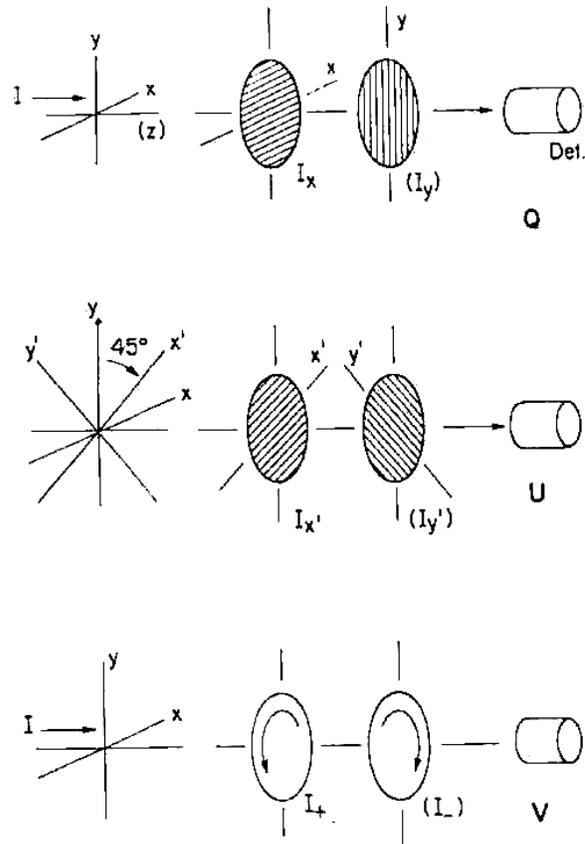


FIG. 2

For Q, we measure the intensity of the beam transmitted through a linear polarizer with passing axis along x ; secondly with the polarizer axis along y . Q is the difference between these fluxes. For U, we do the same thing but with the passing axes first along the $+45^\circ x'$ axis, then along the -45° axis (y'). For V, which measures the circular polarization in the beam, we use two "circular polarizers", which transmit only left-rotating (positive) and right-rotating (negative) circularly polarized components respectively; V is the difference between these two transmitted intensities, measured alternately. (Here a "circular polarizer" is normally a sandwich of a quarter-wave plate followed by a linear polarizer with passing axis at 45° relative to the wave-plate fast axis.)

That the fourfold set I,Q,U,V is at least necessary to define the polarization is obvious. Could we dispense with the second "linear" parameter, U? No, because the beam could be linearly polarized along one of the $\pm 45^\circ$ axes ($x'y'$); the Q measurement would be neutral to this, yielding $Q = 0$, while the beam is indeed polarized.

Let us calculate the Stokes parameters for the (elliptically polarized) wave \mathbf{E} of eq. (1). We can omit the $\exp(-i\omega t)$ time factor since it doesn't affect the intensities. We have:

$$I = |\mathbf{a}|^2 + |\mathbf{b}|^2 = a_1^2 + a_2^2 + b_1^2 + b_2^2 \quad (5)$$

and

$$Q = |\mathbf{a}|^2 - |\mathbf{b}|^2 = a_1^2 + a_2^2 - b_1^2 - b_2^2 \quad (6)$$

For U, we use the projections of \mathbf{E} along the $\pm 45^\circ$ axes $x'y'$, defined by unit vectors:

$$\mathbf{i}' = (\mathbf{i} + \mathbf{j})/\sqrt{2} \quad \text{and} \quad \mathbf{j}' = (-\mathbf{i} + \mathbf{j})/\sqrt{2}$$

$$\text{Thus } E_{x'} = (a + b)/\sqrt{2} \quad \text{and} \quad E_{y'} = (-a + b)/\sqrt{2}$$

and

$$U = |E_{x'}|^2 - |E_{y'}|^2 = ab^* + a^*b = 2(a_1b_1 + a_2b_2) \quad (7)$$

For the circular parameter V, note, that an arbitrary E of type (1) can always be written as a sum of two purely circular E fields of types:

$$\mathbf{e}_+ = (\mathbf{i} + i\mathbf{j})/\sqrt{2} \quad \mathbf{e}_- = (\mathbf{i} - i\mathbf{j})/\sqrt{2} \quad (8)$$

To show this we write

$$\mathbf{E} = a\mathbf{i} + b\mathbf{j} = A(\mathbf{i} + i\mathbf{j})/\sqrt{2} + B(\mathbf{i} - i\mathbf{j})/\sqrt{2} \quad (9)$$

where A and B are two complex constants to be determined. Equating \mathbf{i} and \mathbf{j} coefficients we get:

$$a = (A + B)/\sqrt{2} \quad \text{and} \quad b = i(A - B)/\sqrt{2}$$

yielding $A = (a - ib)/\sqrt{2}$ and $B = (a + ib)/\sqrt{2}$. By definition here, V is the difference in intensities between the A and B circular amplitudes:

$$V = |A|^2 - |B|^2 = i(ab^* - a^*b) = 2(a_1b_2 - a_2b_1) \quad (10)$$

Summarizing (5), (6), (7) and (10) we have:

$$a_1^2 + a_2^2 + b_1^2 + b_2^2 = I$$

$$a_1^2 + a_2^2 - b_1^2 - b_2^2 = Q$$

$$2(a_1b_1 + a_2b_2) = U$$

$$2(a_1b_2 - a_2b_1) = V \quad (11)$$

Now, given the measured values $IQUV$, these equations form a simultaneous set which can be solved for the real E-field amplitudes $a_1b_1a_2b_2$. The solution is unique, except for certain signs: eqs. (11) being quadratic, we can for example change all the signs of the a 's and b 's and get an equally correct solution. This merely means that our intensity measures cannot tell \mathbf{E} from $-\mathbf{E}$. More generally there is an undetermined optical phase. Apart from such phase (and sign) uncertainty, however, eqs. (11) suggest that the four Stokes parameters are sufficient to define the light beam's polarized character, insofar as we can determine this from intensity measurements through polarizers and wave plates.

C. A Word on Jones and Mueller Matrices.

These are two other mathematical formalisms for dealing with polarized light and its interaction with devices. (A reference is Hecht and Zajac, Optics, Addison-Wesley Publ. Co., 1979, pp; 268-271.) Actually, the Jones method is essentially the one I use in this exposition, except for a different notation. In the Jones notation, our complex vector of type $\mathbf{E} = a\mathbf{i} + b\mathbf{j}$ is written as a column matrix:

$$\mathbf{E} = \begin{pmatrix} a \\ b \end{pmatrix}$$

Any optical device acting on \mathbf{E} is then represented by a 2x2 matrix which multiplies (transforms) the column vector \mathbf{E} .

In the Mueller method, one works not with the (complex) \mathbf{E} vector but directly with the Stokes parameters, which are represented by a column matrix or "vector":

$$\mathbf{E} \rightarrow \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

A device acting on \mathbf{E} , then corresponds to a 4x4 matrix (the Mueller matrix) which multiplies (transforms) the column Stokes vector.

Since there are no fundamental advantages in either of these methods, I will use instead merely our complex vectors, except for one example at the end of this treatise.

D. Polychromatic, incoherent, or partially coherent light: A key theorem.

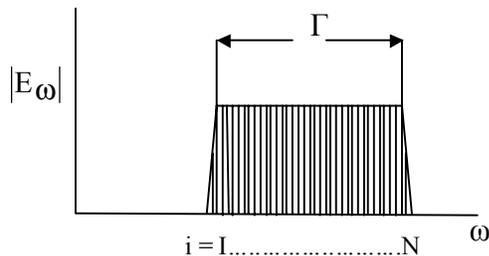


FIG. 3

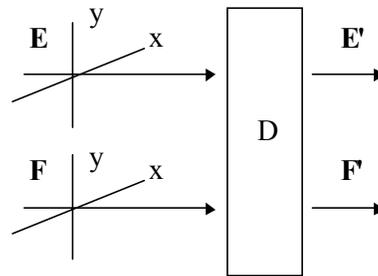


FIG. 4

Suppose we have a distribution of monochromatic light components (**E** vectors) spread over a band of frequency width Γ :

$$\mathbf{E} = \sum_i^N (a_i \mathbf{i} + b_i \mathbf{j}) e^{-i\omega_i t} \quad (12)$$

Since the different frequency components do not interact (in time averages), we can calculate Stokes parameters of **E** quite simply; for example

$$I = \sum_i \left(|a_i|^2 + |b_i|^2 \right) \text{ and so on for } Q, U, V.$$

Now pass the beam **E** through an arbitrary, linear optical device D. Each frequency component $\mathbf{E}_i = a_i \mathbf{i} + b_i \mathbf{j}$ will undergo a linear transformation of the x, y components as follows:

$$\begin{aligned} a_i \mathbf{i} &\rightarrow a_i (\alpha \mathbf{i} + \beta \mathbf{j}) \\ b_i \mathbf{j} &\rightarrow b_i (\gamma \mathbf{i} + \delta \mathbf{j}) \end{aligned} \quad (13)$$

where $\alpha, \beta, \gamma, \delta$ are (complex) constants of the device. They cause attenuation, dichroism, phase shifts, etc. We assume only that the $\alpha, \beta, \gamma, \delta$ are frequency-independent over the bandwidth Γ .

Now consider a second incident beam,

$$\mathbf{F} = \sum_j (A_j \mathbf{i} + B_j \mathbf{j}) e^{-i\omega_j t}$$

which happens to have the same Stokes parameters as **E**, although perhaps a different detailed structure.

After passage through D, the two beams would be transformed by (13):

$$\mathbf{E} \rightarrow \mathbf{E}' \quad \text{and} \quad \mathbf{F} \rightarrow \mathbf{F}'.$$

Now the question is: Are the Stokes parameters of **E'** and **F'** still equal? Straightforward calculation of the transformed parameters uses (5), (6), (7) and (10), applied to:

$$\mathbf{E}' = \sum_{\mathbf{i}} \left(a_i' \mathbf{i} + b_i' \mathbf{j} \right) e^{-i\omega_i t}$$

where the D-transformed amplitudes a_i' and b_i' are given, in light of (13), by

$$a_i' = a_i \alpha + b_i \gamma \quad \text{and} \quad b_i' = a_i \beta + b_i \delta$$

Analogous expressions apply for the alternate field \mathbf{F} and its D-transformed amplitudes A_i' and B_i' . We thus compute expressions for the D-transformed Stokes parameters I_E', Q_E', U_E', V_E' , and I_F', Q_F', U_F', V_F' , in terms of the incident-beam amplitudes a_i, b_i and A_i, B_i . Using the initial assumption that

$$I_E = I_F, Q_E = Q_F, U_E = U_F, V_E = V_F,$$

it can then be shown -- with a bit of algebra -- that $I_E' = I_F', Q_E' = Q_F', U_E' = U_F', V_E' = V_F'$. I leave the detailed algebra to the reader. This result has two important corollaries:

(a) No optical device D can distinguish between different beams in the band having the same Stokes parameters. (Wavelength-dependent devices are excluded.)

(b) Any beam can be represented by an unpolarized component ($Q=U=V=0$), plus a single monochromatic, coherent, polarized wave of type

$$\mathbf{E} = (a\mathbf{i} + b\mathbf{j})e^{-i\omega t}.$$

The ratio between the intensity $|\mathbf{E}|^2$ of this polarized component, and the total intensity, is the "fractional polarization" of the beam.

E. Partial linear polarization. Consider a beam with \mathbf{E} field

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$$

where \mathbf{E}_0 is an unpolarized part and \mathbf{E}_1 is linearly polarized along the direction θ , as

in Fig. 5. The unpolarized part \mathbf{E}_0 in Fig. 5 is represented schematically by a random array of vector fields of different frequencies, directions, phases, etc. In this case we can write

$$\mathbf{E}_1 = E_1(\mathbf{i} \cos \theta + \mathbf{j} \sin \theta)e^{-i\omega_1 t},$$

where we can take the amplitude E_1 to be real.
Clearly:

$$I = |\mathbf{E}_0|^2 + E_1^2$$

and

$$Q = E_1^2(\cos^2 \theta - \sin^2 \theta) = E_1^2 \cos(2\theta).$$

To find U we project \mathbf{E}_1 onto the $\pm 45^\circ$ axes:

$$\begin{aligned} U &= E_1^2 \left| \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) \cdot (\mathbf{i} \cos \theta + \mathbf{j} \sin \theta) \right|^2 - \left| \frac{1}{\sqrt{2}}(-\mathbf{i} + \mathbf{j}) \cdot (\mathbf{i} \cos \theta + \mathbf{j} \sin \theta) \right|^2 \\ &= E_1^2 \sin(2\theta) \end{aligned}$$

The total linearly polarized intensity (flux) is defined as $Q^2 + U^2 = E_1^2$. This would be merely the Stokes parameter Q'' which we would measure if we used not the xy axes, but axes $x''y''$ where x'' is along the vector \underline{E}_1 . The fractional linear polarization is defined by:

$$p = \frac{\sqrt{Q^2 + U^2}}{I} = \frac{E_1^2}{|\mathbf{E}_0|^2 + E_1^2}$$

and we can write

$$Q = I p \cos(2\theta) \text{ and } U = I p \sin(2\theta) \quad (14)$$

Describing a partially linearly polarized beam by the three parameters (I, p, θ) , rather than (I, Q, U) , is often useful.

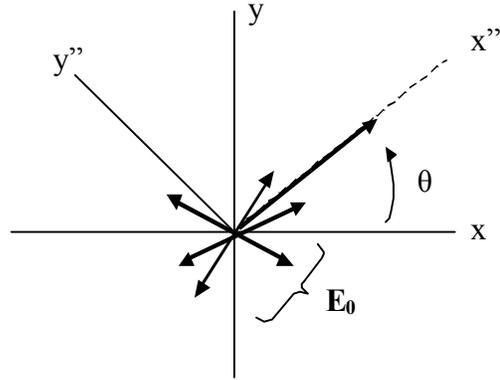


FIG. 5

Rotated axes. Suppose we want the linear Stokes parameters referred not to x, y but to some arbitrarily rotated axes x', y' as in Fig. 6. Simply:

$$Q' = I\rho \cos[2(\theta - \alpha)]$$

$$U' = I\rho \sin[2(\theta - \alpha)].$$

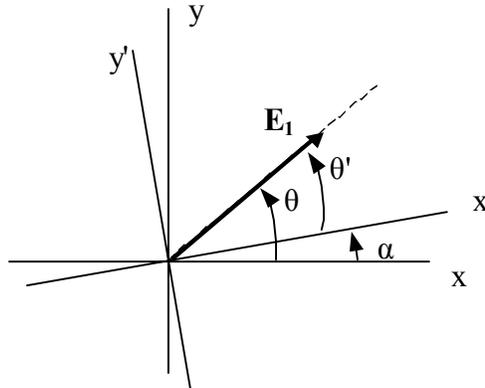


FIG. 6

Expanding the cos, sin functions and using (14) we get an important linear transformation:

$$Q' = Q \cos(2\alpha) + U \sin(2\alpha)$$

$$U' = -Q \sin(2\alpha) + U \cos(2\alpha) \quad (15)$$

F. Measuring and modulating polarized light with birefringence modulators.

We refer to devices which involve time-varying birefringence: The two transverse refractive indices n_x, n_y vary in time, and more specifically the birefringence $n_x - n_y$ varies. This causes a varying optical phase shift $\Delta\phi$ between the transmitted wave components E_x and E_y :

$$\Delta\phi(t) = \frac{2\pi d}{\lambda} (n_x - n_y) = \frac{2\pi d}{\lambda} f(t)$$

where d is the device thickness and λ is the light wavelength. Such devices include photoelastic modulators (PEMs) and Pockels cells. In many applications the modulating

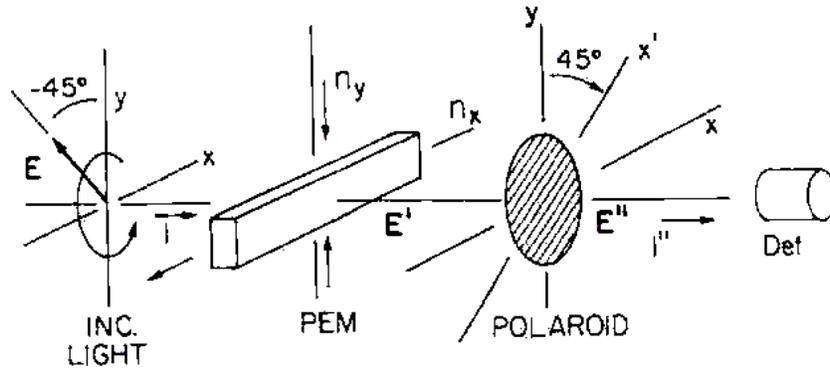


FIG. 7

function $f(t) = n_x - n_y$ has zero time average, and is a periodic function. In the case of Pockels cells, the modulating function may be arbitrary; squarewave modulation, for example, is often used. PEMs, however, are resonant devices and $f(t)$ is sinusoidal. We will treat cases of PEM modulation.

In Fig. 7 is shown a general set-up for measuring the polarization of light, using a photoelastic modulator -- PEM. The incident beam has a polarized component with the general form

$$\mathbf{E} = (a\mathbf{i} + b\mathbf{j}) \quad (\text{omitting the } e^{-i\omega t} \text{ factor})$$

which is elliptically polarized, with a, b complex. This can represent a completely polarized beam or merely the polarized part of a partly polarized, partly coherent beam as described in Secs. D and E above. The unpolarized part would not be modulated by the PEM. The wave \mathbf{E}' after the PEM can be written:

$$\mathbf{E}' = a\mathbf{i}e^{iA/2} + b\mathbf{j}e^{-iA/2}$$

where $A = A_0 \cos \Omega t = \Delta \phi(t)$; A_0 is the retardation amplitude, which is determined by the excitation level of the PEM. The wave \mathbf{E}'' after the Polaroid in Fig. 7, which has its passing axis at 45° , is gotten by projecting \mathbf{E}' onto the 45° (x') axis using the unit vector along that axis:

$$\mathbf{E}'' = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) \cdot \mathbf{E}' \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$$

and the output intensity is:

$$I'' = \frac{1}{2} \left| a e^{iA/2} + b e^{-iA/2} \right|^2 \quad (16)$$

Take the case $\mathbf{E} = (\mathbf{i} - \mathbf{j})/\sqrt{2}$, i.e. $a = 1/\sqrt{2}$ and $b = -1/\sqrt{2}$, an incident beam linearly polarized along -45° . This gives:

$$\begin{aligned} I'' &= \frac{1}{4} |2i \sin A/2|^2 = \sin^2(A/2) = (1 - \cos A)/2 \\ &= [1 - \cos(A_0 \cos \Omega t)]/2 \end{aligned} \quad (17)$$

We can learn something about the intensity function (17) by supposing A_0 to be small, and expanding the cosine function. Thus if $A_0 \ll 1$, we find:

$$I'' \cong \left(A_0^2/8 \right) [1 - \cos(2\Omega t)] \quad (18)$$

This modulation is second harmonic, at angular frequency 2Ω . (The PEM modulation period is $T = 2\pi/\Omega$.) For values of A_0 not small, the waveforms of I'' look as in Fig. 8 at below. In the waveform 2 we have $A_0 = \pi/2$, "half wave" retardation amplitude. The general expansion of eq. (17) involves Bessel-function coefficients:

$$I'' = [1 - J_0(A_0) + 2J_2(A_0)\cos(2\Omega t) - 2J_4(A_0)\cos(4\Omega t) + \dots]/2 \quad (19)$$

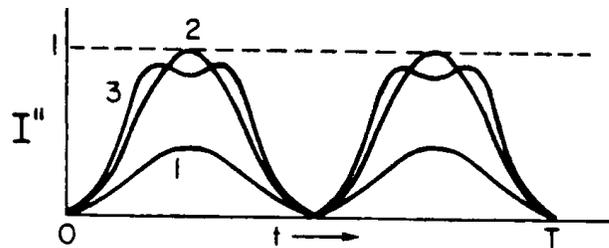


FIG. 8

Usually the second harmonic is used for synchronous detection of the linearly polarized flux here. In (19), J_2 has a maximum of 0.486 with $A_0 = 3.1$, or approximately $\pi = 3.14$ (half wave retardation).

Partial polarization, calibration. Suppose in the above case the initial beam is partly polarized, with an unpolarized intensity I_0 and a polarized part of intensity $I = E^2$. The total intensity is $I_t = I_0 + I$. How do we find the fractional polarization $p = I/I_t$ from the synchronous second-harmonic signal? In this case the signal received would be:

$$S = I J_2(A_0) \cos(2\Omega t)$$

We can calibrate this signal by placing a (second) Polaroid across the beam to the left of the PEM in Fig. 7, rendering the beam totally polarized. If we align this Polaroid along -45° , the polarized I signal here will not be affected, but an additional output signal will arise proportional to $I_0/2$ -- note that only half of the I_0 flux is transmitted. Thus the total signal with the Polaroid calibrator is:

$$S_c = (I + I_0/2)J_2(A_0)\cos(2\Omega t)$$

and the fractional polarization is found from the ratio S/S_c :

$$p = \frac{S/S_c}{2 - S/S_c} \quad (20)$$

This formula assumes a perfect calibrator Polaroid with no insertion loss for light polarized along its passing axis. For precision, a correction factor for the calibrator should be applied. Note that for small p , roughly $p = (S/S_c) / 2$.

The above measurement applied to light linearly polarized along one direction, here at 45° to the PEM axis. To characterize a beam with an arbitrary, unknown polarization direction we could make two measurements with two orientations of the PEM and analyzer- polarizer in Fig. 7, 45° apart, yielding effectively the two linear Stokes parameters Q and U (Sec. B). More directly this would give the two normalized parameters Q/I and U/I .

Consider now how the set-up of Fig. 7 can sense circular polarization. In this case we take the incident beam to have a right-circular component

$$\mathbf{E}'' = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) \quad \text{for which } a = 1/\sqrt{2} \text{ and } b = i/\sqrt{2}.$$

Putting this into (16) gives:

$$I'' = 1/2 - 1/2 \sin(A_0 \cos \Omega t)$$

Again a standard expansion gives:

$$I'' = 1/2 - J_1(A_0)\cos(\Omega t) + J_3(A_0)\cos(3\Omega t) - \dots \quad (21)$$

Circular polarization gives a leading signal at the fundamental (first harmonic) PEM frequency. In (21), J_1 has a maximum of about 0.58 at $A_0 = 1.85$, slightly more than “quarter wave” operation in which case $A_0 = \pi/2 = 1.57$.

As with linear polarization, if in this case the beam is only partly circularly polarized we need a form of calibration to find the fractional polarization. Here this can be done by placing a "circular polarizer" in the incident beam. (This can be made as a sandwich of a linear polarizer and a $\lambda/4$ plate.) By comparing the signals analogous to S and S_c above (without and with the calibrator respectively), we can get the fractional circular polarization V/I .

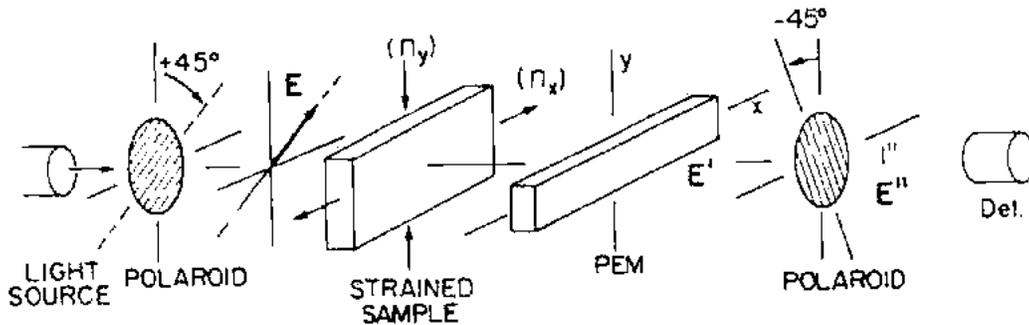


FIG. 9

G. Birefringence and strain measurements. In the set-up of Fig. 9 we use crossed Polaroids oriented at $\pm 45^\circ$, and an intervening PEM. A birefringent sample is placed between the first Polaroid and the PEM, with its birefringence axes (fast/slow axes) along x, y . We have in mind here either naturally birefringent samples (some crystals), or mechanically strained optical parts, in which birefringence is induced by the piezo-optical effect. Let the birefringent phase shift between the x and y light components be B (in radians). A unit-amplitude field \underline{E} after the first Polaroid is:

$$\underline{E} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$$

and the field \underline{E}' after the PEM, modified by the sample and PEM phase shifts, is:

$$\underline{E}' = \left[\mathbf{i}e^{i(B+A)/2} + \mathbf{j}e^{-i(B+A)/2} \right] / \sqrt{2} \quad (22)$$

where again, $A = A_0 \cos(\Omega t)$ is the PEM retardation function. To get the output intensity I'' we must project \mathbf{E}' onto the -45° second-Polaroid passing axis, yielding:

$$\begin{aligned} E'' &= \frac{1}{\sqrt{2}} (\mathbf{i} - \mathbf{j}) \cdot \mathbf{E}' = \frac{1}{\sqrt{2}} \left[e^{i(B+A)/2} - e^{-i(B+A)/2} \right] \\ &= \sqrt{2} i \sin[(B+A)/2] \end{aligned}$$

and the output intensity seen by the detector is:

$$\begin{aligned} I'' &= |E''|^2 = 2 \sin^2[(B+A)/2] = 1 - \cos(B+A) \\ &= 1 - \cos(B)\cos(A) + \sin(B)\sin(A) \end{aligned} \quad (23)$$

The cosine term here contains only even harmonics of the PEM frequency Ω , while the sine term is made of odd harmonics. Usually (though perhaps not always), the sample birefringence B is very small, $B \ll 1$. In this case (23) simplifies to:

$$\begin{aligned} I'' &\cong (1 - \cos A) + B \sin A = (1 - \cos A) + B \sin(A_0 \cos \Omega t) \\ &= 1 - \cos(A_0 \cos \Omega t) + 2 B J_1(A_0) \cos(\Omega t) - \dots \text{ third and higher terms.} \end{aligned}$$

Here the term $\cos(A_0 \cos \Omega t)$ is merely the usual term in second and higher even harmonics that we have seen above in linear-polarization measurement; to first order it is not affected by the birefringent sample. The term proportional to B , the $\cos \Omega t$ first-harmonic term, is the source of the signal used for synchronous detection of the birefringence -- using a lock-in detector, for example. As in circular polarimetry (Sec. F above), we would optimize $J_1(A_0)$ by setting the retardation amplitude A_0 to about 1.85, roughly speaking "quarter wave" operation.

Physically, the birefringent sample renders the linearly polarized light, after the first Polaroid, partly circularly polarized. This circular component then produces a first-harmonic signal in the detector just as in the normal detection of circular polarization - eq. (21) above.

To get the actual value of the birefringent retardation B , in radians, a simple way is to replace the "sample" in Fig. 9 with a quarter-wave plate. In that case, $B = \pi/2$ and $\sin(B) = 1$, in the exact equation (23). Then, for the case of small sample birefringence,

$$B \equiv \frac{\text{1st harm. signal with sample}}{\text{1st harm. signal with } \lambda/4 \text{ plate}} \quad (24)$$

(This doesn't take into account "insertion losses" of the sample and/or quarter wave plate. Those should be found from transmittance measurements and a correction factor applied to (24) as needed.)

Strain birefringence and strain amplitude. A uniaxial strain s -- a fractional compression or distension of a material along some direction -- results, in a transparent material, in a fractional birefringence given by:

$$\frac{n_x - n_y}{n_0} = \eta s$$

where n_0 is the unstrained refractive index and η is a piezo-optical coefficient characteristic of the material. Both s and η here are dimensionless. Typically, η tends to be a fraction of order of unity, such as 0.2. We can find the strain from the birefringent retardation given by (24), by writing:

$$B = \frac{2\pi t}{\lambda} (n_x - n_y) = \frac{2\pi t}{\lambda} n_0 \eta s \quad (25)$$

where t is the sample thickness and λ is the wavelength.

As a numerical example here, consider the parameters $t = 1$ cm, $\lambda = 1$ micron, $n_0 = 1.5$, and $\eta = 0.2$. Then roughly, $s \sim 10^{-4} B$. Since B as small as 10^{-3} radian can usually be detected with a PEM set-up, strains as small as 10^{-7} can be sensed.

The actual sign of a measured birefringence must sometimes be found by an empirical test. In ordinary glasses, a squeezing stress produces a fast axis (smaller refractive index) along the squeeze direction. Thus a C-clamped glass block can be used as a sample to find the true signs of the B signals as in the apparatus of Fig. 9. Or, a commercial $\lambda/4$ plate with known fast axis can be used.

H. Example of an unusual polarization technique: Shifting a light beam's frequency.

Here I will deal with a special topic which is interesting in itself but which also provides good examples of polarized-light transformations through devices. Suppose we would like to shift the frequency of a light beam. This would normally be of interest with monochromatic laser light. Applications might include optical Doppler radar and related

technologies. At first glance such shifting might seem impossible -- except by reflecting the beam from a moving object. But consider the set-up in Fig. 10:

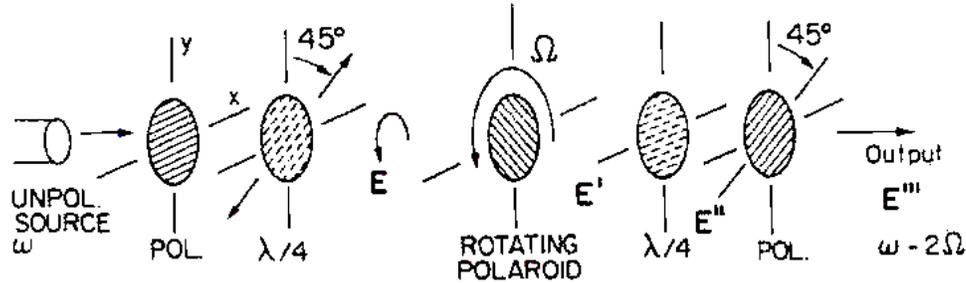


FIG. 10

Unpolarized light of angular frequency ω is passed through a Polaroid and $\lambda/4$ plate to produce circularly polarized light:

$$\mathbf{E} = (\mathbf{i} + i\mathbf{j})e^{-i\omega t}$$

We pass this light through a Polaroid which is mechanically rotated at angular frequency Ω , producing a "rotating" linearly polarized field. To find the transmitted field \mathbf{E}' we project \mathbf{E} onto a rotating unit vector along the rotating Polaroid's passing axis:

$$\begin{aligned} \mathbf{E}' &= [(\mathbf{i} + i\mathbf{j}) \cdot (\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t)](\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t)e^{-i\omega t} \\ &= (\cos \Omega t + i \sin \Omega t)(\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t)e^{-i\omega t} \\ &= (\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t)e^{-i(\omega - \Omega)t} \end{aligned} \quad (26)$$

(Note: In this section, for economy I am throwing away $1/\sqrt{2}$ factors in unit vectors.)

Next we pass this beam through a $\lambda/4$ plate with fast axis along x , which causes a (relative) optical phase shift of $\pi/2$ between the x and y components. Since $\exp(i\pi/2) = i$, effectively this gives:

$$\mathbf{E}'' = (\mathbf{i} \cos \Omega t + i\mathbf{j} \sin \Omega t)e^{-i(\omega - \Omega)t}$$

Finally we pass this through a Polaroid passing along the 45° direction $(\mathbf{i} + \mathbf{j})$, yielding:

$$\begin{aligned} \mathbf{E}''' &= (\cos \Omega t + i \sin \Omega t) e^{-i(\omega - \Omega)t} (\mathbf{i} + \mathbf{j}) \\ &= e^{-i(\omega - 2\Omega)t} (\mathbf{i} + \mathbf{j}) \end{aligned} \quad (27)$$

The output beam with \mathbf{E}''' here is a pure monochromatic beam with shifted angular frequency $\omega - 2\Omega$. An upshifted frequency $\omega + 2\Omega$ could of course be obtained by reversing the rotating Polaroid direction or the waveplate polarity. Physically, the rotating Polaroid "subtracts", or "adds", cycles per second, to the optical field oscillation. It is surprising that a pure frequency shift results here, without the generation of certain unwanted sidebands.

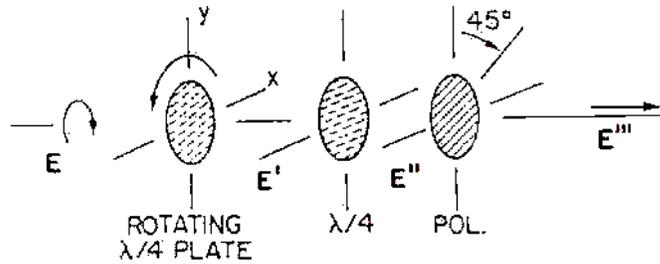


FIG. 11

A rotating wave plate, rather than a rotating polarizer, also can be used for frequency shifting. In Fig. 11 we show incident light which has first been rendered circular, i.e.

$$\mathbf{E} = (\mathbf{i} + i\mathbf{j}) e^{-i\omega t}$$

In Fig. 11, the first $\lambda/4$ plate rotates at angular frequency Ω , and has its fast and slow axes along the (rotating) directions x' and y' respectively. The x' , y' axes are along (rotating) unit vectors $\mathbf{i}' = \mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t$ and $\mathbf{j}' = -\mathbf{i} \sin \Omega t + \mathbf{j} \cos \Omega t$. We project the field \mathbf{E} onto \mathbf{i}' and \mathbf{j}' , and impose a relative phase shift of $\pi/2$ between x' , y' components caused by the wave plate. The phase shift amounts to multiplying the y' component by $\exp(i\pi/2) = i$. Thus the field beyond the rotating wave plate can be written:

$$\begin{aligned} \mathbf{E}' &= \{[(\mathbf{i} + i\mathbf{j}) \cdot (\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t)](\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t) \\ &\quad + i[(\mathbf{i} + i\mathbf{j}) \cdot (-\mathbf{i} \sin \Omega t + \mathbf{j} \cos \Omega t)](-\mathbf{i} \sin \Omega t + \mathbf{j} \cos \Omega t)\} e^{-i\omega t} \end{aligned}$$

This boils down to:

$$\mathbf{E}' = [\mathbf{i} (\cos \Omega t + \sin \Omega t) + \mathbf{j} (\sin \Omega t - \cos \Omega t)] e^{-i(\omega - \Omega)t} \quad (28)$$

Next we add a stationary $\lambda/4$ plate (see Fig. 11), with axes along x, y . This again imposes a factor of $\exp(i\pi/2) = i$, on the y or \mathbf{j} component in (28), yielding:

$$\mathbf{E}'' = [\mathbf{i} (\cos \Omega t + \sin \Omega t) + i\mathbf{j} (\sin \Omega t - \cos \Omega t)] e^{-i(\omega - \Omega)t}$$

Finally a Polaroid at 45° then passes an output wave of amplitude:

$$\begin{aligned} \mathbf{E}''' &= \mathbf{E}'' \cdot (\mathbf{i} + \mathbf{j}) / \sqrt{2} \\ &= (1/\sqrt{2}) [(\cos \Omega t + \sin \Omega t) + i(\sin \Omega t - \cos \Omega t)] e^{-i(\omega - \Omega)t} \\ &= (1/\sqrt{2}) \left(e^{i\Omega t} - i e^{i\Omega t} \right) e^{-i(\omega - \Omega)t} = (1/\sqrt{2}) (1 - i) e^{-i(\omega - 2\Omega)t} \end{aligned} \quad (29)$$

The factor $(1-i)$ here amounts merely to an unimportant $\pi/4$ phase shift. The output wave given by (29) has a pure frequency shift of 2Ω , as with the rotating Polaroid arrangement.

Physical picture of the frequency shift. The rotating $\lambda/4$ plate of Fig. 11, acting on the incident circularly polarized beam, produces a rotating linear polarization, described by a vector field

$$\mathbf{E} = (\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t) e^{-i\omega t}$$

It is useful here to think of a rapidly oscillating but slowly rotating \mathbf{E} field, with $\Omega \ll \omega$. Now, such a rotating, linearly polarized field can be decomposed into two contrarotating, circularly polarized fields \mathbf{E}_+ and \mathbf{E}_- , with different frequencies. In Fig. 12, we show in (a) the two rotating components just prior to their cross-over which produces a net maximum field \mathbf{E} . Suppose the field \mathbf{E}_- to have a higher frequency than the field \mathbf{E}_+ . At a slightly later time but (after many cycles of the rapid optical oscillation at angular frequency ω , we show in (b) an instant just before coincidence of the fields \mathbf{E}_- and \mathbf{E}_+ produces

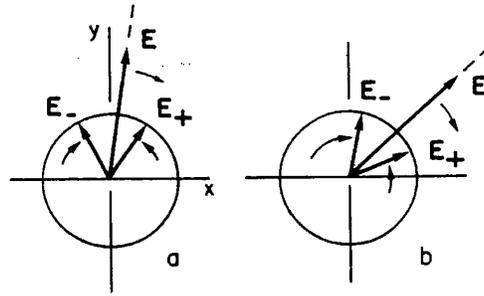


FIG. 12

another maximum. Since the E_- field rotates somewhat faster, this maximum will occur at an angle rotated somewhat in the clockwise direction. Thus the net E field, composed of these contra-rotating components, will itself rotate -- in this case clockwise -- at the (low) angular frequency Ω . Formally we can decompose the rotating "linear" field E into contra-rotating circular components as follows:

$$\mathbf{E} = (\mathbf{i} \cos \Omega t + \mathbf{j} \sin \Omega t) e^{-i\omega t} = (1/2) \left[(\mathbf{i} + i\mathbf{j}) e^{-i(\omega+\Omega)t} + (\mathbf{i} - i\mathbf{j}) e^{-i(\omega-\Omega)t} \right]$$

If we extract one of these contrarotating circular components, using as in Fig. 11 the "circular polarizer" combination of $\lambda/4$ wave plate plus Polaroid, we will obtain a pure upshifted or downshifted frequency component.

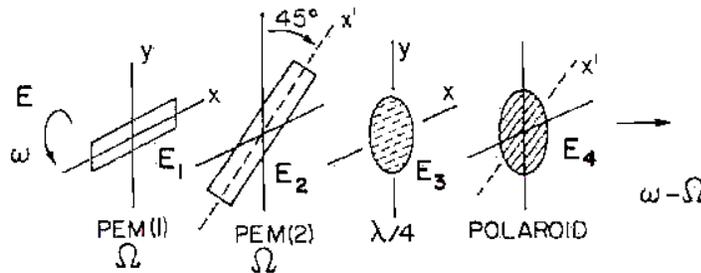


FIG. 13

Frequency shifting with a PEM pair. The just-described schemes for frequency shifting require mechanically rotating components, and realistic shifts would be limited to a few hundred Hertz or so. Could PEMs or Pockels cells be used in this context, permitting much higher shifts and avoiding moving parts? The answer is yes, although there is a minor qualification in that the resulting shifted beam cannot be quite spectrally pure.

Consider a PEM pair as in Fig. 13. The two units are mounted with their axes 45° separated; the first has its axis along x , the second along the $45^\circ(x')$ axis. The PEMs

have identical modulation frequencies $f = \omega/2\pi$, and are slaved together, with a 90° electrical phase shift imposed on one relative to the other. (With commercial PEMs of the Hinds type this would be accomplished by permitting one of them to self-oscillate; a slaving signal derived from that one would be used to drive the other unit, with a 90° phase shift interposed electronically.) Such a PEM pair, with a quadrature phase relationship between the excitations, acts essentially like a rotating wave plate: The fast axis is, for example, first along x , then along x' , then along y , and so on. The properties of the arrangement in Fig. 13 can be worked out using the methods of this monograph, as follows.

Let the incident wave be circularly polarized, $\mathbf{E} = (\mathbf{i} + \mathbf{j})e^{-i\omega t}$ (see eq. (3) above). (With an unpolarized or linearly polarized source beam, as from a laser, a $\lambda/4$ plate and a Polaroid or merely a $\lambda/4$ plate would be added at the left in Fig. 13, as needed.) The PEM retardation functions are written:

$$A = A_0 \cos(\Omega t) \quad \text{and} \quad B = A_0 \sin(\Omega t) \quad (30)$$

Here we assume the two amplitudes to be the same, but the amplitude is adjustable. The wave after the first PEM (dropping the $\exp(-i\omega t)$ factor) is:

$$\mathbf{E}_1 = \left(\mathbf{i}e^{iA/2} + \mathbf{j}e^{-iA/2} \right) / \sqrt{2} \quad (31)$$

To find the wave after the second PEM, we project \mathbf{E}_1 onto the $\pm 45^\circ$ directions x' and y' , using unit vectors $\mathbf{i}' = (\mathbf{i} + \mathbf{j})/\sqrt{2}$ and $\mathbf{j}' = (-\mathbf{i} + \mathbf{j})/\sqrt{2}$, as in the above sections; we impose modulated phase factors $\exp(\pm iB/2)$ on the x' and y' components respectively. After a modest amount of algebra we find that the wave can be written:

$$\mathbf{E}_2 = \left\{ \left[\cos(A/2 - \pi/4)e^{i(B/2 + \pi/4)} + \cos(A/2 + \pi/4)e^{-i(B/2 + \pi/4)} \right] \mathbf{i} \right. \\ \left. + \left[\cos(A/2 - \pi/4)e^{i(B/2 + \pi/4)} - \cos(A/2 + \pi/4)e^{-i(B/2 + \pi/4)} \right] \mathbf{j} \right\} / (2\sqrt{2}) \quad (32)$$

(The reader may arrive at different but equivalent forms for this expression.)

In Fig. 13, the $\lambda/4$ plate and the Polaroid combination act as a circular polarizer, with the eigenaxes of these components rotated 45° , as used also in the arrangements of Figs. 10 and 11. They serve to extract one circular component from the wave \mathbf{E}_2 . In particular, we want to extract the "negative" circular component of type $(\mathbf{i} - \mathbf{j})$, which rotates oppositely from the incident wave $\mathbf{E} = (\mathbf{i} + \mathbf{j})$. Let us do this directly in this case

by noting that the wave \mathbf{E}_2 , as any complex vector, can be written as the sum of two circular components:

$$\mathbf{E}_2 = R(\mathbf{i} + i\mathbf{j})/\sqrt{2} + S(\mathbf{i} - i\mathbf{j})/\sqrt{2}$$

where R and S are to be determined. We want the S part, which will be just the output amplitude, i.e. $E_4 = S$. Obviously:

$$S = E_4 = \mathbf{E}_2 \cdot (\mathbf{i} + i\mathbf{j})/\sqrt{2} \quad (33)$$

Performing the operation (33) and rearranging we find, after some work:

$$E_4 = (-1/4)\{(1-i)\sin[(A+B)/2] - (1+i)\sin[(A-B)/2]\} \quad (34)$$

Now we insert the PEM excitation functions $A = A_0 \cos(\Omega t)$ and $B = A_0 \sin(\Omega t)$, yielding:

$$E_4 = (-1/4)\{(1-i)\sin[(A_0/\sqrt{2})\cos(\Omega t - \pi/4)] + (1+i)\sin[(A_0/\sqrt{2})\sin(\Omega t - \pi/4)]\} \quad (35)$$

The two sine functions in (35) can be expanded in harmonic terms with Bessel- function coefficients. After some manipulation we obtain:

$$E_4 = (i/\sqrt{2})\left[J_1(A_0/\sqrt{2})e^{i\Omega t} + J_3(A_0/\sqrt{2})e^{i3\Omega t} + \dots \right] \quad (36)$$

If we re-insert the optical time factor $\exp(-i\omega t)$, we can write:

$$E_4 = (i/\sqrt{2})\left[J_1(A_0/\sqrt{2})e^{-i(\omega-\Omega)t} + J_3(A_0/\sqrt{2})e^{-i(\omega-3\Omega)t} + \dots \right] \quad (37)$$

This exhibits an output beam consisting entirely of downshifted components. The leading term has a maximum for $A_0/\sqrt{2} = 1.85$, or $A_0 = 2.62$ (radians): and $J_1(1.85) = 0.581$. For this choice of A_0 , note that the second term in (37), the third-harmonic term, is proportional to $J_3(1.85) = 0.10$. Thus the ratio of intensities of these two terms, which are given by the squares, is about 34: 1. Higher harmonics (5Ω , etc.) are absolutely negligible.

The spectral purity of the beam shifted to $\omega - \Omega$, with the maximum-transmission case $A_0/\sqrt{2} = 1.85$, is therefore around 96%. Higher purity can be attained with a slight

loss of transmission, by reducing A_0 somewhat. For $A_0/\sqrt{2} = 1.0$, the intensity ratio for the J_1 and J_3 terms in (37) is about 500, and the spectral purity exceeds 99%. Note that the unshifted component, of optical frequency ω , is totally blocked, i.e. absent -- presuming, of course, that we have perfect $\lambda/4$ plates and linear polarizers.

As with the rotating Polarizer of $\lambda/4$ plate, here we can induce an upshifted rather than a downshifted frequency merely by changing the sense of rotation -- in this case by using a -90° phase shift, rather than $+90^\circ$, between the two PEM excitations.

The reader may have noticed that the rotating polarizer and wave plate arrangements produce 2Ω frequency shifts, whereas the PEM pair produces a simple Ω shift. This merely reflects the fact that with a physically rotating $\lambda/4$ plate, the fast axis rotates geometrically twice as fast as in the case of the PEM pair, if the rotation and modulation frequencies are defined as in the above arrangements.

PEM-pair frequency shifting: Jones-calculus analysis.

As we said in Sec. C above, we have avoided the use of matrix methods in this treatise. But as an illustration of such methods for those readers familiar with matrix algebra, we will outline here the analysis of one example, that of frequency shifting with a PEM pair as just treated above. We use the Jones method.

In the Jones notation, a vector field $\mathbf{E} = (a\mathbf{i} + b\mathbf{j}) \exp(-i\omega t)$, or simply $a\mathbf{i} + b\mathbf{j}$ omitting the optical time variation, is represented by a column matrix with elements a, b . The notation assumes a set of axes, such as x, y . An optical component then induces a transformation of \mathbf{E} expressed by a 2×2 matrix -- the Jones matrix -- which acts on the (a, b) column matrix, producing another column matrix (a', b') by matrix multiplication. In Fig. 13, the first PEM is represented by a matrix, written in x, y coordinates, as:

$$M_1 = \begin{pmatrix} e^{iA/2} & 0 \\ 0 & e^{-iA/2} \end{pmatrix} \quad (38)$$

The second PEM in Fig. 13 is rotated 45° ; its eigen axes are the axes x', y' . To deal with this we first project any given vector (a, b) onto x', y' coordinates, using a projection or rotation matrix; we apply the second PEM's transformation, analogous to (38), in the x', y' frame; then project (rotate) the transformed components back to the x, y system. The resulting Jones matrix for PEM(2), in x, y coordinates, is:

$$M_2 = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} e^{iB/2} & 0 \\ 0 & e^{iB/2} \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \quad (39)$$

$$= \begin{pmatrix} \cos(B/2) & i \sin(B/2) \\ i \sin(B/2) & \cos(B/2) \end{pmatrix} \quad (40)$$

The next item in the optical train of Fig. 13 is a $\lambda/4$ plate, which imposes a 90° optical phase shift between the x and y components. This has the Jones matrix:

$$M_3 = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$

The last item is a Polaroid with its passing axis at a 45° angle. Written in x', y' coordinates this has a simple Jones matrix which merely "passes" any x' component but nullifies any y' component:

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (41)$$

For this to act on a vector expressed in x, y components we must first transform the vector into x', y' components; then act on it in the x', y' system using (41). Thus the effect of the final Polaroid is expressed by the matrix:

$$M_4 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 \end{pmatrix} \quad (42)$$

Note that this last transformation "leaves us" in the x', y' system, rather than in the x, y system. In this case that is all right since the only surviving vector component is along the x' passing axis of the final Polaroid, and we are interested only in its intensity and time variation. The entire optical train of Fig. 13 is thus effectively represented by the matrix product:

$$(M) = (M_4)(M_3)(M_2)(M_1) \quad (43)$$

In this case the input is, specifically, the circularly polarized beam $\mathbf{E} = \mathbf{i} + \mathbf{ij}$. The output is a field linearly polarized along $+45^\circ$, of form $\mathbf{E}' = a'\mathbf{i}'$. In the Jones notation,

$$\begin{pmatrix} a' \\ 0 \end{pmatrix} = (M_4)(M_3)(M_2)(M_1)\begin{pmatrix} 1 \\ i \end{pmatrix} \quad (44)$$

By carrying out the matrix multiplications in (44), the reader will find that this yields a component a' which is essentially just the output field amplitude E_4 of equation (36).

1. A note on PEM-pair combinations: Beat-frequency pairs, etc.

A variety of special modulation arrangements are possible with a pair of PEMs (or analogous modulators), used in tandem. In some cases the PEMs can be co-aligned; in others, as in the scheme in Fig. 13 above, they are oriented with a 45° separation around the light axis. The two PEMs can be driven at the same frequency -- with or without an electrical phase shift between them -- or they can run at different, normally incommensurate, frequencies. The properties of all such PEMs pairs can be worked out using the methods of this monograph, following a scheme essentially like that just used above for the PEM pair in the frequency-shifting application.

Such PEM pairs have various applications. Some examples:

(a) Low-frequency, heterodyne detection, of polarized light. Sometimes the inherently high modulation frequencies of PEM devices are a hindrance. This is especially true for measurement of the polarization of infrared light; many sensitive detectors for the middle infrared, such as bolometers and cooled solid-state detectors, are slow, with high-frequency roll-offs as low as a few hertz. In the simple "polarimeter" set-up of Fig. 7, the single PEM can be replaced by a PEM pair, with colinear axes and with different angular frequencies Ω_1 , and Ω_2 . If both PEMs are driven at $\lambda/4$ modulation, incident light linearly polarized along the 45° axis produces a heterodyne signal in the detector at angular frequency $\Omega_1 - \Omega_2$. The beat frequency can in principle be very low, e.g. a few hertz, although for a reasonably stable beat frequency the stability of the PEM frequencies is a limitation. Since PEMs of the Hinds type are "free running" oscillators of frequencies determined by the mechanical self-resonance of the device, a reference signal for lock-in detection in this case would be gotten by mixing the two PEM reference signals electronically and extracting the difference-frequency signal.

Circular polarization can be detected with such a beat-frequency PEM pair; a signal of angular frequency $2\Omega_1 - \Omega_2$, proportional to the circularly polarized component in the incident light, is generated in the detector as in Fig. 7. By making Ω_1 close to half Ω_2 , a low-frequency signal can be obtained in this case also.

(b) Simultaneous detection of Q and U linear polarizations. As noted on page 13, second-harmonic detection of linearly polarized light using the single-PEM set-up of Figure 7 gives only one of the two linear Stokes parameters, specifically the component polarized at 45° to the PEM axis. With that simple set-up, to measure the other component -- and thus to completely characterize the linear polarization -- we would have to rotate the PEM (and analyzer Polaroid) by 45° . However, we can do this without "moving parts" by using a PEM pair: The two PEMs are oriented 45° apart around the light axis, and have different frequencies f_1 and f_2 . These could be comparable but well-separated frequencies, e.g. 50 kHz and 51 Khz. The analyzer Polaroid is placed with passing axis half way between the two PEM axes, i.e. 22.5° away from either PEM axis. It is easy to show that detector signals at the two frequencies f_1 and f_2 , proportional to Q and U components respectively, are both generated. These can be read simultaneously in two lock-in detectors referenced separately to the two PEMs. The signals are half as strong as the simple single- PEM signal of Fig. 7. It can be shown that the PEMs do not interact; the Q and U signals each faithfully reflect the independent Q and U light components.

(c) Modulated frequency shifting. In the set-up of Figure 13, for shifting the frequency of a light beam with a PEM pair, the analysis supposed the two PEMs to be driven coherently at the same frequency Ω , yielding a simple constant frequency shift $\omega \rightarrow \omega \pm \Omega$. But if the PEM frequencies are different, say $\Omega_1 = \Omega + \alpha/2$ and $\Omega_2 = \Omega - \alpha/2$ where α is a small difference frequency, then it can be shown that the output light frequency oscillates back and forth between $\omega + \Omega$ and $\omega - \Omega$, at the (low) angular beat frequency α . Such an FM modulation of a light beam could be used for example in a kind of "derivative spectroscopy", producing a detector signal proportional to the derivative $dI/d\lambda$ of light with a spectrum $I(\lambda)$.

J. A few references.

The author is an astronomer, and I have used and developed PEM-based polarimeters in astronomical polarimetry. There is a large literature on polarized light instrumentation and application in fields ranging from chemistry (circular dichroism, fluorescence, etc.) to mineralogy (optical crystallography), with which I am not in touch. Much such work involves modulators such as PEMs or Pockels cells.

Two general reference books are:

W.A. Shurcliff, Polarized Light, Harvard University Press, 1962.

D. Clarke and J.E. Grainger, Polarized Light and Optical Measurement, Pergamon Press, Oxford, 1971.

From my own work I list the following papers:

J.C. Kemp, "Piezo-optical birefringence modulators: New use for a long-known effect", *J. Opt. Soc. Am.*, 59, 950 (1970). A basic description of the PEM.

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J.C. Kemp, "Photoelastic-modulator polarimeters in astronomy", *Proc. Soc. Photo- Opt. Instr. Engrs. (SPIE)*, 307, 83 (1981). Has a general discussion of types of PEMs.

J.C. Kemp, G.D. Henson, C.T. Steiner, I.S. Beardsley, and E.R. Powell, "The optical polarization of the Sun measured at a sensitivity of parts in ten million", *Nature*, 326, 270 (1987). Describes some state-of-the-art measurements using PEM instruments; includes description of a heterodyne (double PEM) system for a special purpose.