BIREFRINGENCE MEASUREMENT

APPLICATION NOTE

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A material with linear birefringence has different refractive indices for (orthogonal) linear polarized light beams passing through it. Birefringence is an intrinsic material property. The integrated effect along a light path passing through a bulk sample is called retardation or retardance.

Figure 1 depicts a birefringent sample with the retardation axes marked as x and y for a light beam normal to the sample surface.

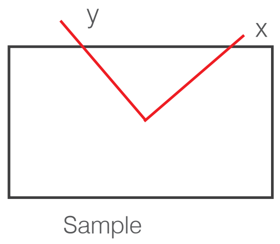

Figure 1: Birefringent sample with refractive indices marked

The two refractive indices are designated as nx and ny. If nx < ny then the x axis is called the "fast axis" and the y axis is called the "slow axis" of the sample. The retardation of the sample therefore must be described by two parameters: a magnitude and a direction. The orientation of the fast axis is generally used to describe the direction of the retardation.

The magnitude of the retardation (in units of length) is given by

$$R(nanometers) = (n_y - n_x)d(nanometers) \qquad (1)$$

where d is the thickness of the sample. For many uses in optics, the retardation is required as a phase angle, related to the light wavelength. Conversion to phase angle may be made by Equations 2 and 3.

$$R(nanometers) = \frac{2\pi R(nanometers)}{\lambda(nanometers)} \qquad (2)$$

or

$$R(degrees) = \frac{360R(nanometers)}{\lambda(nanometers)} \qquad (3)$$

STRESS BIREFRINGENCE

Isotropic materials such as glass do not exhibit birefringence unless the material is subjected to stress. The stress may be externally induced or it may be internal stress, induced in the manufacture of the material sample. The photoelastic modulator (PEM) offers a very sensitive method of measuring low levels of retardation arising from stress in an optical material sample.

MEASURING RETARDATION WITH A PEM

An optical bench setup that may be used for measuring the retardation of birefringent samples is shown in Figure 2. Use of a vertical optical bench setup is recommended since the sample may be supported on a shelf with a hole for the light to pass through.

Use of a laser for the light source simplifies the optical setup since no collimating and focusing lenses need be used.[1] The

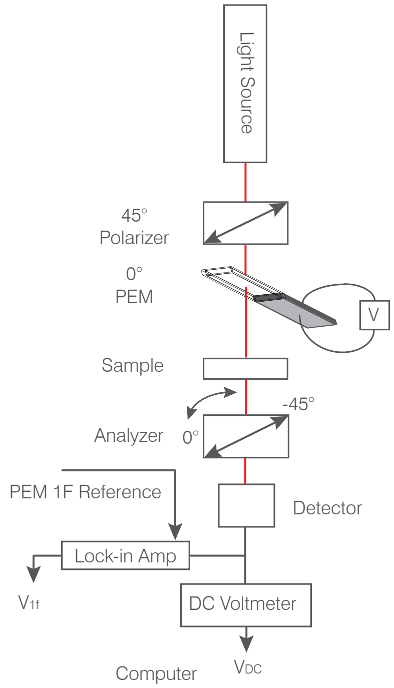

Figure 2. A diagram for basic setup using the PEM for sequentially measuring data from both set-ups I and II where the analyzer is oriented at 0° or -45°

laser is followed by a polarizer oriented at +45° with respect to the PEM retardation axis. The PEM retardation axis defines 0° for the coordinate system of the setup.

The modulated light passes through the sample to a second polarizer, then to a detector. The detector output is split with one branch going to a lock-in amplifier for detection of the 2f AC signal and the other branch going to a DC voltmeter. Use of a low-pass filter before the voltmeter is advisable. A Hinds signal conditioning unit will provide low-pass filtering and amplification of the DC signal.

Two different orientations of the second polarizer are indicated: at –45° (Setup I) and at 0° (Setup II). For setup I the retardation component at 0° (and 90°) is measured. (Reference 1) For setup II the retardation component at +45° (and -45°) is measured. (Reference 2)

For optimum performance the PEM retardation should be set to R = 0.383 waves = 2.405 radians. For this retardation setting the "average" or DC voltage is constant, independent of the orientation of the second polarizer, the sample orientation and the sample retardation.

Two signals are detected: 1) a lock-in amplifier detects the rms voltage V_{1f} of the detector signal at the PEM frequency 1f and 2) a digital voltmeter or other device records the average voltage or V_{DC} of the signal. It is convenient to form the ratio of V_{1f} to V_{DC}. (Equation 2) The quantity R is insensitive to fluctuations in the light source intensity, changes in optical transmission, etc.

$$R_{1f} = \frac{V_{1f}}{V_{DC}} \qquad (4)$$

The optical setups may be analyzed by using the Mueller calculus. The signal amplitudes given in these analyses are in terms of peak voltages or voltage amplitudes. Thus a correction factor of √2 will appear to convert from one quantity to another. PEM theory shows that

$$\sqrt{2}R_{1f} = 2J_1(A)\sin\delta \qquad (5)$$

Where δ is the sample retardation in angle units (e.g. radians), $J_1(A)$ is a Bessel function of the PEM retardation A and R_{1f} is the ratio of the 1f signal amplitude measured by the lock-in amplifier and the average voltage (V_{DC}). Equation 5 is valid for both setup I and setup II.

Therefore

$$\sin\delta = \frac{R_{1f}}{\sqrt{2}J_1(A)} \qquad (6)$$

or

$$\delta = \sin^{-1}\left[\frac{R_{1f}}{\sqrt{2}J_1(A)}\right] \qquad (7)$$

DETERMINING RETARDATION MAGNITUDE AND FAST AXIS ORIENTATION

There are two methods of determining retardation magnitude and fast axis orientation.

1. Mount the sample on a rotation stage. Rotate the sample until the ratio R_{1f} is a maximum. The retardation.magnitude is given by equation 7. The fast axis of the sample is parallel to the PEM retardation axis.
2. With the sample in a fixed orientation
   a. Measure the retardation using setup 1 (second polarizer at -45°) Call the retardation for this measurement δ_1.
   b. Rotate the second polarizer to 0°. (Setup II) Measure the retardation and call the result δ_{II}.
   c. Calculate the magnitude of the retardation from
   $$\delta = \sqrt{\delta_I^2 + \delta_{II}^2} \qquad (8)$$
   d. Calculate the fast axis angle from
   $$\rho = \frac{1}{2}\tan^{-1}\frac{\delta_{II}}{\delta_I} \qquad (9)$$

ADVANCED TECHNIQUES

Hinds Exicor® systems are designed for rapid measurement of retardation magnitude and fast axis direction. They utilized a beam-splitting mirror and two polarizer/detector assemblies, one for Setup I and one for Setup II. (Reference 3)

Equations 6 and 7 work well for small values of the retardation δ (e.g. $\delta << \pi/2$). For retardation $\delta \approx \pi/2$ radians $= \lambda/4$ waves the accuracy becomes poor. Reference 4 gives techniques for measurements in these cases.

Exicor 300AT

REFERENCES

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3. Wang and Oakberg, A new instrument for measuring both the magnitude and angle of low level linear birefringence, RevSciInstr 70, 10 pp 3847-3854, October 1999.

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