HINDS INSTRUMENTS™

Polarization Metrology of Anisotropic Materials

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Full Mueller matrix measurements allow for elimination of ambiguity and a fuller understanding of polarization metrology. The Exicor® 150XT system offers complete Mueller matrix measurement from 350nm to 800nm with automated sample translation and optional tip-tilt measurements. Integration times as low as 0.01 seconds for measurements of the Mueller matrix to 0.001. Partial Mueller matrix systems can attain measurements of 8 Mueller parameters with precision of 0.0001.

Polarization metrology covers a broad range of techniques from ellipsometry, circular dichroism, and transmission polarimetry. Each technique focuses on the measurement and modeling of a subset of possible polarimetric interactions of a sample. Mueller matrix polarimetry in both transmission and reflection can address the needs of a broad range of applications with a single system, and unite a range of different metrology techniques under a single banner.

The polarization state of light can be expressed as a four element Stokes vector, [*I,Q,U,V*]. To express the interaction of a Stokes vector with a sample, the 4x4 Mueller matrix contains all of the polarimetric optical properties of a sample: linear retardance, linear extinction, circular retardance and circular extinction as well as information about depolarization present in the sample.

The anisotropy of aligned molecules leads to linear optical properties such as linear retardance and linear extinction. The measurement of linear retardance has been applied to the measurement of internal strain of a material. By measuring the linear retardance of a sample, and relating that measured retardance through the photoelastic constant of a material, the internal stresses from injection molding, annealing and curing can be measured. In reflection, linear extinction and retardance can be modeled and fit to find the refractive index of layered materials and now composes the broad field of ellipsometry.

As an example of a material that exhibits both linear extinction, and linear retardance, Figure 1 shows the measured Mueller matrix for a petrographic

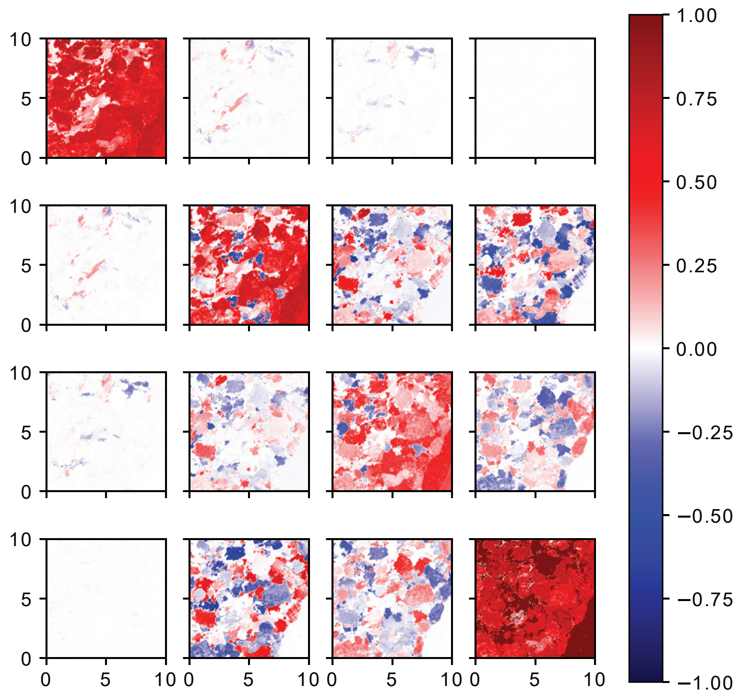

Figure 1: Mueller matrix of petrographic thin section at 633nm using a Hinds Exicor 150XT transmission Mueller polarimeter.

thin section at 633nm using a Hinds Exicor 150XT transmission Mueller polarimeter with optional imaging objective able to achieve resolutions of 30um or better. The quartz crystals in the sample exhibit only linear retardance while the linear extinction is likely due to inclusions of corundum or tourmaline.

The measured Mueller matrix can be reduced into linear optical properties; absolute linear extinction, alignment of the linear extinction axis, absolute linear retardance and alignment of the linear retardance.

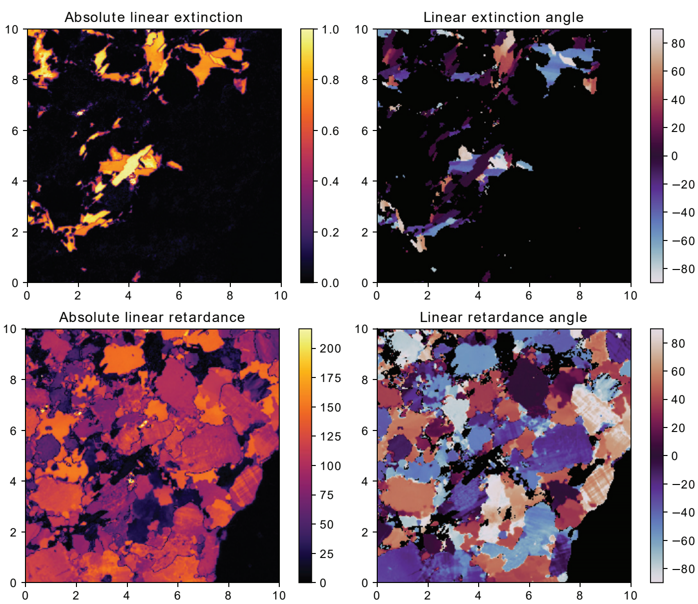

Figure 2: Linear extinction and linear retardance measured of petrographic thin section at 633nm using a Hinds Exicor 150XT transmission Mueller polarimeter.

If the sample contains a homogeneous material with a single optical property, the Mueller matrix elements can be directly related to their respective optical properties as shown below.

$$M \propto \begin{bmatrix} T & LE & LE' & CE \\ LE & T & CR & -LR' \\ LE' & -CR & T & LR \\ CE & LR' & -LR & T \end{bmatrix}$$

In the equation above, T is the transmission, LE is the linear extinction along 0°/90°, LE' is the linear extinction along 45°/135°, LR and LR' are similarly the linear retardance, CE is the circular extinction, and CR is the circular retardance. The diagonal elements, T, are more complicated than shown here. For materials with multiple optical properties, this simple relation does not hold.

A more accurate description of the Mueller matrix can be formed from the differential Mueller matrix, m.

$$M = e^{-m}$$

$$m = \ln -M = \begin{bmatrix} 0 & LD & LD' & -CD \\ LD & 0 & -CB & -LB' \\ LD' & CB & 0 & LB \\ -CD & LB' & -LB & 0 \end{bmatrix}$$

The notations change to capture the difference in going from extrinsic properties of a sample to the theoretically intrinsic properties of a material in transmission. The physical meaning of these quantities only holds in transmission, but for reflection measurements, this procedure serves to minimize the measurable quantities from 16 matrix elements into 6 minimal properties. For a more complete treatment see Arteaga or Ossikovski.

The circular polarization properties of circular retardance and circular extinction arise in a material from the more subtle effects of symmetry. They are orders of magnitude smaller than their linear counterparts. The simple measurement of circular extinction in solution has a broad range of applications in biology, but the measurement of these minute changes in polarization is challenging in solid samples where the effects are dwarfed and mimicked by the much larger linear optical properties.

Linear optical properties often are used to generate pseudo circular optical properties. The common circular polarizer is composed of two layers; a linear retarder and a linear polarizer. The action of these two elements together generates a strong pseudo-circular polarizer, but this effect is completely dependent on the order of the optical components.

The strength of full Mueller matrix metrology lies in the ability not just to measure the polarization properties of a sample, but to examine the effects of layered structures and elicit the origin of the signal of interest. For instance, the layering of a linear retarder and linear polarizer will inevitably lead to an asymmetric Mueller matrix.
For an instrument such as a standard circular dichroism spectrometer, the Mueller matrix element M03 is measured and converted into a millidegrees. For solid samples, any minuscule linear extinction arising from scattering, dichroism, or actual circular dichroism is measured. For a Mueller matrix measurement, asymmetry between M30 and M03 acts as an indication of the origin of the measured circular dichroism signal.

The measurement of circular retardance is likewise impacted by the presence of linear retardance. Only in known single crystalline samples can the optical rotation be practically attributed to material properties with confidence. As an example of such a case, common single crystal quartz exhibits strong optical rotation along the axis of symmetry (c-cut). Shown in Figure 3 is the measured Mueller matrix of a c-cut quartz crystal at 633nm using a Hinds Instruments Exicor 150XT Mueller polarimeter. The instrument included a tip-tilt stage to enable 2-axis rotation as opposed to translation.

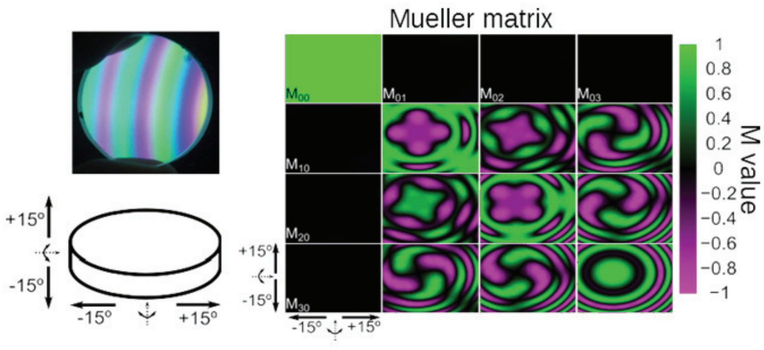

Figure 3: Mueller matrix of c-cut quartz at 633nm using a Hinds Exicor 150XT transmission Mueller polarimeter.

This Mueller matrix is then reduced into measured linear and circular retardance. The measurable circular retardance wraps every time the system's total retardance peaks at a full wave. This wrapping is an unavoidable consequence of phase measurements, and the measured quantities need to be unwrapped. Shown in Figure 4 are the measured circular, linear and total retardance.

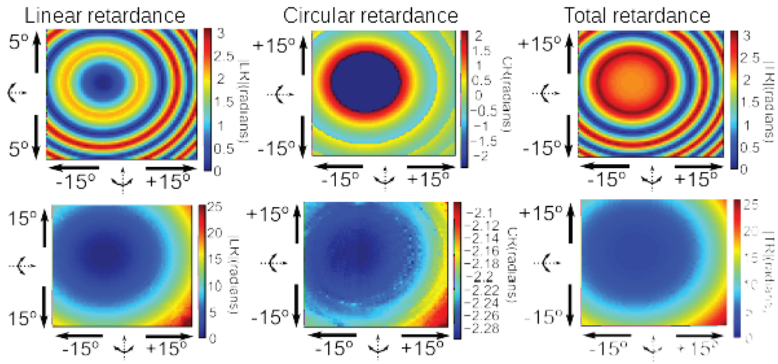

Figure 4: Absolute linear retardance, circular retardance, and total retardance of c-cut quartz at 633nm using a Hinds Exicor 150XT transmission Mueller polarimeter.

These quantities can then be reduced to their tensor components given the thickness of the sample, or simply fit to find the same quantities. For a more complete treatment see Arteaga or Gupta.

The Mueller matrix contains all of these optical properties as well as quantization of depolarization. Most polarization metrology systems measure only a subset of the Mueller matrix and may face challenges understanding the origin of measured optical properties.

Measuring the complete Mueller matrix requires generating and analyzing both linear and circular polarization. This can accomplished without moving parts by utilizing four photoelastic modulators as seen in Arteaga. The use of photoelastic modulators offers a wide spectral range, fast modulation, and stable operation. The basic layout of a four PEM Mueller polarimeter is given in Figure 5, below.

The magnitude of each Mueller matrix element is a scaled amplitude of different harmonics of the each

PEM. For instance, the element M02 is measured at the sum of the first, and second modulator.

Conclusion

By measure the amplitude of PEM frequencies, the Hinds Exicor 150XT transmission Mueller polarimeter can measure the complete Mueller matrix or a range of samples. The full Mueller matrix allows for a more robust understanding of the sample's polarimetric properties. Particularly in solid samples, the various optical properties will mix and complicate the analysis of any single optical property.

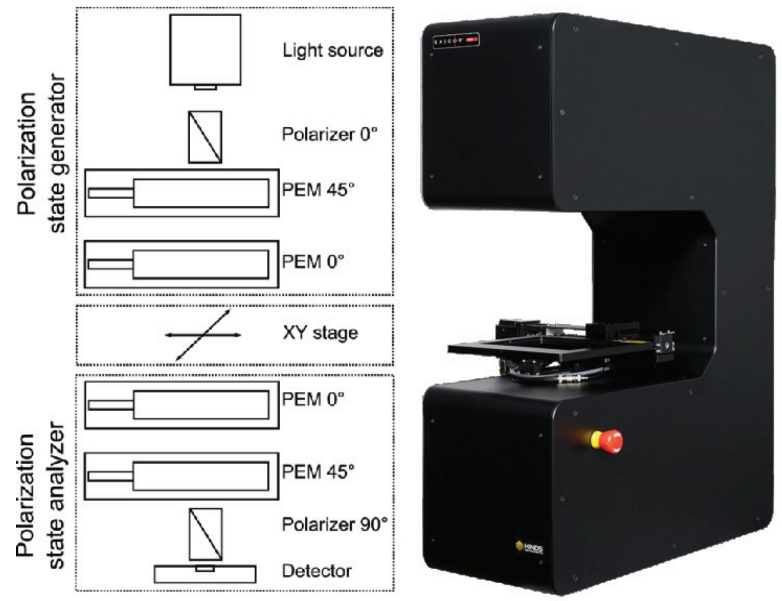

Figure 5: Hinds Exicor 150XT transmission Mueller polarimeter.

References

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